Use of Ballistic Arcs in Low Thrust Navigation

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A technique is presented for improving navigation accuracy in the Solar Electric Encke Slow Flyby Mission proposed to encounter the comet Encke during its 1980 apparition. The effect of the dominant navigation error source, the high-level thruster noise, is reduced through the introduction of a ballistic coast arc for the purpose of orbit determination enhancement. The placement and duration of the arc is investigated with respect to its impact on final delivery error and on control effort required for trajectory correction. The relative contributions of the various individual navigation error sources to the total delivery error and the relative effectiveness of various data type combinations are analyzed and compared for a navigation strategy which includes a coast arc and one which does not. Also examined are the changes in delivery accuracy and control effort as a result of the time lag between the state estimation process and the thrust control program update.

I. Introduction

RECENTLY there has been growing interest in the possible use of solar electric propulsion (SEP) for cometary exploration. A strong candidate for the first mission is a slow flyby of the short-period comet Encke during its 1980 apparition.[1] This flight would serve as a precursor to a rendezvous with Encke in 1984, and would have among its objectives: 1) The determination of the existence of a cometary nucleus. 2) The collection of data on the composition and physical processes of the cometary environment. 3) The demonstration of solar electric propulsion capabilities. 4) The demonstration of operational procedures for SEP control and navigation.

Preliminary mission design calls for launch on Jan. 16, 1979, with Encke encounter occurring 660 days later on Nov. 6, 1980. The nominal encounter conditions, dictated by the science requirements, include a 10-day terminal coast with a closest approach of 1000 km at a velocity of 4 km/sec.[2] The aim point and relative velocity are chosen to provide good imaging science and ample time within the cometary environment for a variety of other scientific observations. The coast is imposed in order to prevent contamination of the cometary environment by the thruster exhaust and to prevent electromagnetic interference with the science instruments by the thrust subsystem. Table 1 summarizes the mission parameters, and Fig. 1 describes the overall heliocentric trajectory.

To ensure maximum science return, the nominal encounter conditions must be met to an accuracy (3σ) of 500 km and 1 km/sec. Although the velocity constraint can be satisfied without difficulty, previous analyses have shown that the spacecraft cannot be delivered to the desired position accuracy by a straightforward navigation strategy.[3,4] It is the objective of this paper to investigate a modification of the basic navigation plan, namely, the insertion of a ballistic coast arc, which can be used to enhance navigation accuracy and ultimately to achieve the desired encounter conditions within the prescribed limits.

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Index categories: Navigation, Control, and Guidance Theory; Spacecraft Navigation, Guidance, and Flight-Path Control Systems.

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II. Navigation Errors and Strategy

The terminal navigation period begins 40 days prior to Encke encounter. During this time disturbances arising from a number of error sources cause an unguided spacecraft to deviate from its nominal flight path. To counteract those disturbances, an active navigation strategy employing dual-station radio tracking, onboard optical measurements, and Earth-based comet observations is used to obtain estimates of the trajectory errors and to supply updates to the thrust vector control program. The error sources, data types, observation schedules, and orbit determination and guidance techniques, are as follows.

Navigation Error Sources

Terminal navigation errors are due primarily to uncertainties in the initial spacecraft state (position, velocity, and mass), uncertainties in the comet ephemeris, biases in the thrust vector pointing angles, and random nongravitational accelerations (process noise) arising from a number of sources. The a priori spacecraft state uncertainties, given in Table 2, are representative of accuracy levels obtainable with typical heliocentric cruise navigation strategies. The Encke ephemeris uncertainties employed in this study were obtained from the Computer Sciences Corporation, where, under contract to Goddard Space Flight Center, improvement of the ephemeris is currently under investigation.[5] Reference 6 briefly discusses the ephemeris error analysis procedure, and Fig. 2 gives a time history of the ephemeris uncertainty reduction during the terminal navigation period.

As has been shown in previous studies,[3,11,12] the dominant navigation error source for an SEP spacecraft is the process noise, i.e., the stochastic nongravitational accelerations acting upon the spacecraft. The major contributors to the process noise during powered phases of the mission are temporal variations in the thrust vector magnitude and direction.

Table 1 Encke slow flyby mission parameters

Launch date	Jan. 16, 1979
Launch C_3	58.5 km^2/sec^2
Arrival date	Nov. 6, 1980
Arrival V_{HP}	4.0 km/sec
Initial mass	1781 kg
Delivered mass	1249 kg
Solar power at 1 AU	14.25 kw
End of thrust phase	650 days
Time of flight	660 days
Solar distance at encounter	0.784 AU
Earth distance at encounter	0.333 AU

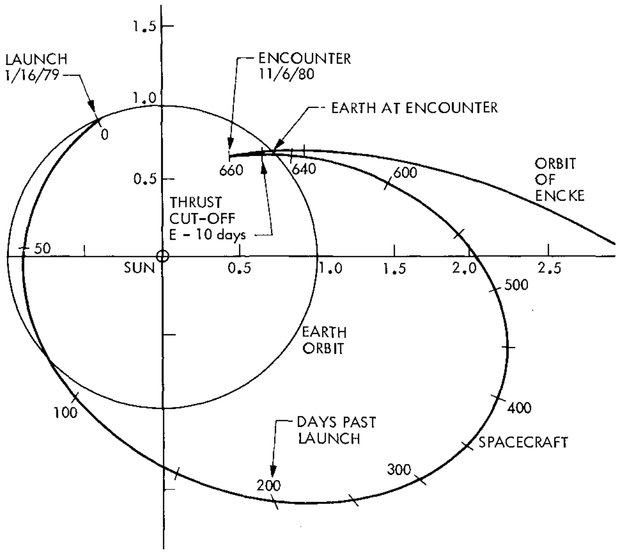

Fig. 1 Encke slow flyby trajectory.

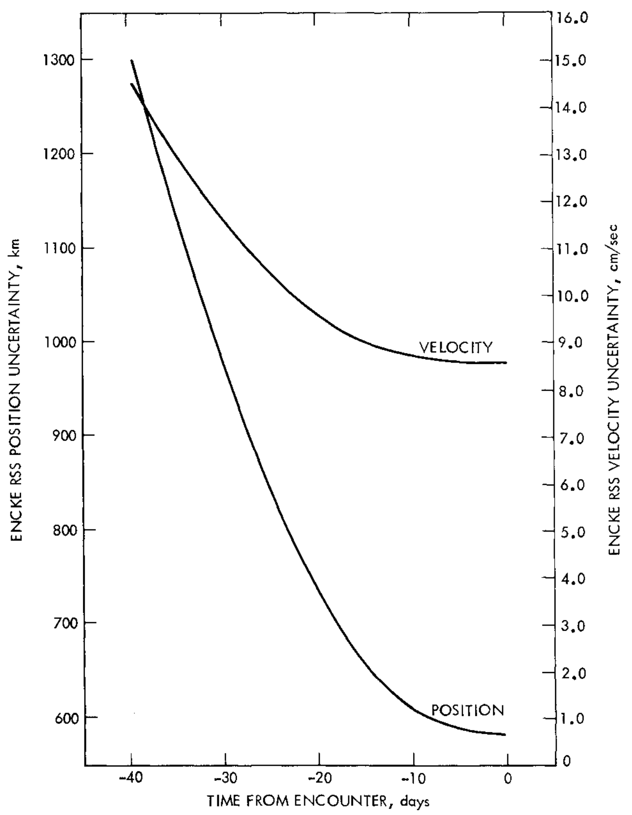

Fig. 2 Prediction of Encke ephemeris uncertainty (referred to 9/27/80).

Recent experience with actual thruster operation indicates that the thrust magnitude variations consist of a relatively small short term fluctuation superimposed on a much larger long term drift. In addition, normal operation of the attitude control system contributes a stochastic component to the thrust vector direction relative to the celestial references. Because of celestial sensor errors and a number of other factors, there may also be a small bias error in the thrust vector direction. Although this bias is not an important error source, it is included for completeness. During the coast phases of the mission, all of the thrust-related noise sources vanish, but other sources must be taken into account, namely, gas leaks from the cold gas attitude control system and errors in the solar pressure model. Solar pressure effects on the SEP spacecraft will be greater than on a ballistic spacecraft because the projected area of the fully extended solar panels will be much larger. Each of the time-varying nongravitational acceleration components is modeled as a first-order Gauss-Markov stochastic process with standard deviations and correlation times given in Table 2.

Table 2 Dynamic error sources

Description	Standard deviation	Correlation time
A priori state errors		
Position	5000 km	...
Velocity	5 m/sec	...
Mass	20 kg	...
Gas leaks + solar pressure accel.	5.0×10^{-12} km/sec^2	1 day
Thrust vector noise		
Short-term instability	0.5% of nominal thrust	1 day
Long-term drift	10.0% of nominal thrust	20 days
Direction temporal variations	15 mrad	1 hr
Direction bias	5 mrad	...

Radio Tracking

The radio tracking strategy consists of a sequence of two-station tracking cycles with the pattern shown in Fig. 3. Because of the high declination of the flight path with respect to the Earth's equatorial plane during the encounter phase of the mission, the spacecraft will rarely be visible to the southern hemisphere, but tracking stations of the northern hemisphere will have very long view periods. Thus, the tracking cycle of Fig. 3 is realizable and a reasonable choice for this phase of the mission.

During each cycle the primary station obtains a full pass of conventional two-way doppler data plus a range point. The secondary station, to the extent that its view period overlaps that of the primary, generates simultaneous two-way doppler data by means of the simultaneous interference tracking technique (SITT) recently developed at JPL and demonstrated with Mariner 10.[7] This technique is preferred over the more conventional methods of obtaining simultaneous data (i.e., simultaneous two-way and three-way data with one station transmitting and two receiving), because it completely eliminates interstation clock synchronization and frequency bias as error sources. The radio data error sources explicitly included in this analysis are random data noise (consistent with the inherent accuracy of the measurements) and tracking station location errors. The effects of the transmission media, Earth polar motion, and Earth spin rate which remain after calibration are implicitly included as equivalent station location errors. Table 3 summarizes the assumptions concerning the radio data error sources. The station location

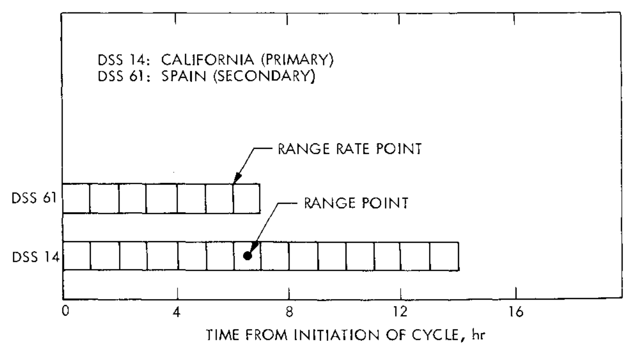

Fig. 3 Simulated radio tracking pattern.

Table 3 Measurement error sources

Description	Standard deviation
Radio data	
DSN station location errors	
Spin radius	1.5 m
Longitude	3.0 m
Data noise	
Range	3.0 m
Range rate (primary station)	1 mm/sec for 1 min sample
Range rate (secondary station)	2 mm/sec for 1 min sample
Optical data	
Camera biases	1 pixel (30 μrad)
Data noise	1 pixel
Earth-based comet data	
Right ascension	3 arc-sec
Declination	3 arc-sec

error component parallel to the spin axis has been omitted because preliminary studies indicated that the navigation analysis for this mission is insensitive to it.

Onboard Optical Data

The optical data consists of a sequence of pictures of the comet against a star background taken with the science imaging subsystem (tentatively, the Mariner 9 narrow angle TV camera). The pictures are taken at 4 hour intervals starting at comet nucleus detection, about 20 days before encounter, and continuing to encounter. Errors in the optical data are due to biases and distortions in the scan platform-TV camera subsystem, image center finding errors, and random measurement noise. The biases and distortions can be calibrated, and it is assumed that the errors remaining after calibration are at the level of a single picture element (pixel). The 1σ values of both the biases and the noise are given in Table 3. Because of the small size of the comet nucleus (1 to 5 km), the effect of the center finding error is not significant and has been omitted from the analysis.

Earth-Based Comet Observations

The Earth-based comet measurements are right ascension and declination astronomical observations made every 10 days from the time of comet recovery (expected to be July 9, 1980) to the time of encounter. For the terminal navigation analysis, the comet ephemeris error covariance is initialized at its predicted value on Sept. 27, 1980 (40 days before encounter), and measurements are taken at 30, 20, and 10 days before encounter. In accordance with Ref. 6, the 1σ error in those measurements is assumed to be 3 sec. of arc.

Orbit Determination and Guidance

The orbit determination filter employed in the analysis is a discrete form of the Kalman sequential filter[8,9] with the additional capability of "considering" the effect of specified parameters without explicitly estimating them. The process noise errors are treated as piecewise constant functions over one-hour time intervals, closely approximating the continuous Gauss-Markov process. The estimated parameters include the spacecraft state and mass, the comet state, the thrust direction angle biases, and the various components of the process noise model given in Table 2. The DSN station locations and the camera biases are considered as error sources by the filter, but are not explicitly estimated.

The guidance algorithm is a discrete linear optimal strategy which minimizes expected terminal position errors by computing variations in the encounter time and piecewise constant updates to the nominal thrust vector magnitude and clock and cone angles.[10] Control corrections are constant over one-day intervals, and the control program is updated daily.

III. Baseline Navigation Analysis

The performance of the navigation system is evaluated with respect to the accuracy to which it can deliver the spacecraft to the desired encounter conditions. The quantitative measure of delivery accuracy is the semimajor axis of the position error ellipse in the impact plane (plane normal to the relative velocity vector at encounter, Fig. 4). Errors in this plane must be corrected by changes in the thrust vector control program, but errors normal to it can be removed by a time-of-flight correction.

The 0.997 probability delivery error ellipse for the baseline navigation strategy is displayed in Fig. 5. Also shown is the ellipse corresponding to the orbit determination error at the start of the science coast mapped to encounter. This ellipse represents the limiting delivery accuracy for an impulsive maneuver performed just prior to the start of the coast. In practice, the limiting delivery accuracy can never be achieved because there is no opportunity to correct the errors due to process noise acting during the final guidance cycle, even though new information concerning these disturbances may be gained from additional data acquired during this time. Since the delivery ellipse has a semimajor axis of 1747 km, the baseline navigation strategy clearly fails to attain the required 500 km accuracy. Moreover, since the mapped orbit determination ellipse has a semimajor axis of 773 km, the 500 km level could not be reached even with a perfect final impulsive maneuver.

To understand the causes of the large delivery error, a breakdown of the contributions of the various error sources is given in Fig. 6. It is apparent that the long term thrust noise produces the largest effect, closely followed by the thrust vector direction noise, the radio data noise, and the camera biases. Further evidence of the effect of the thrust vector noise is seen in the breakdown of the relative effectiveness of the various data types, Fig. 7. Since comet observations of some type, either onboard or Earth-based, are clearly essential for successful navigation, only data combinations including at least one of these data types are evaluated. Conventional radio tracking, known to be very sensitive to process noise, in combination with Earth-based comet observations, gives the poorest performance, as expected. Use of onboard optical measurements gives significant improvement by providing direct comet relative position information. However, the process noise degrades the ability to infer relative velocity from the optical data, and consequently limits delivery accuracy. The addition of dual station tracking (SITT), with its inherently low sensitivity to process noise, provides a remarkable further improvement in performance, but is still unable to attain the desired accuracy level.

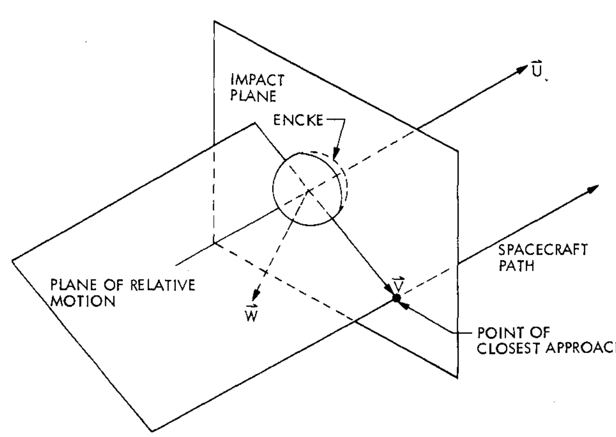

Fig. 4 Impact plane system.

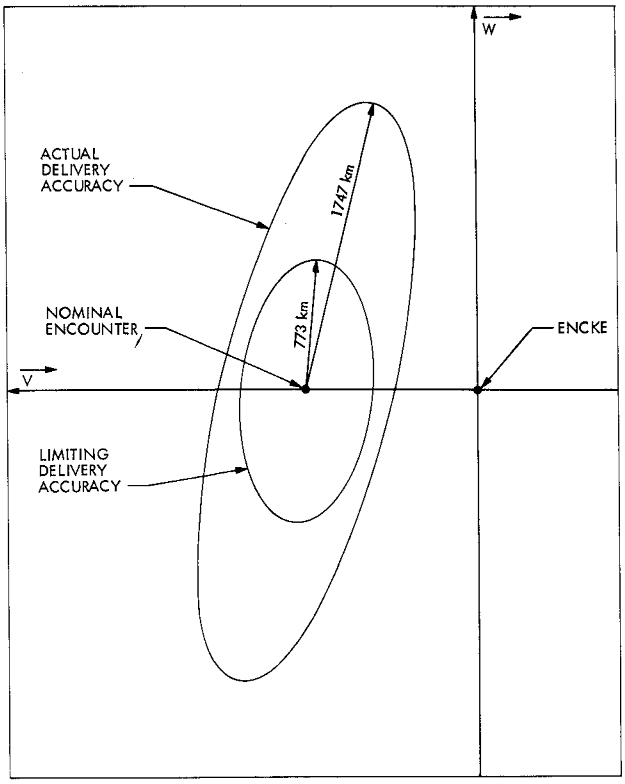

Fig. 5 Delivery error ellipses in the impact plane at encounter for the baseline case.

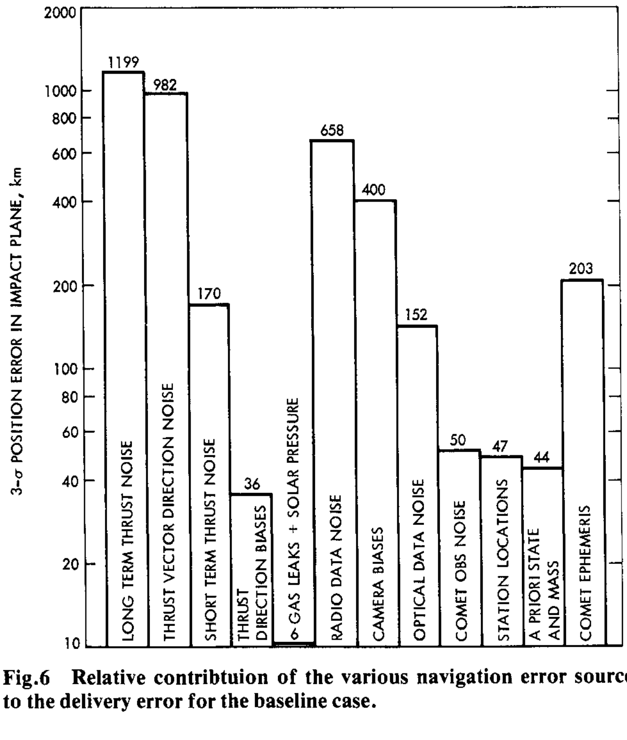

Fig.6 Relative contribtuion of the various navigation error sources to the delivery error for the baseline case.

IV. Ballistic Arc Analysis

Despite the use of multiple data sources and advanced orbit determination and guidance techniques, the baseline navigation strategy is unable to satisfy the required delivery accuracy of 500 km. Since the primary cause of the failure is the thrust vector noise, a straightforward approach to improvement is to shut down the thrusters for a period of time to increase orbit determination accuracy and then to bring them back to briefly to perform a final trajectory correction. To

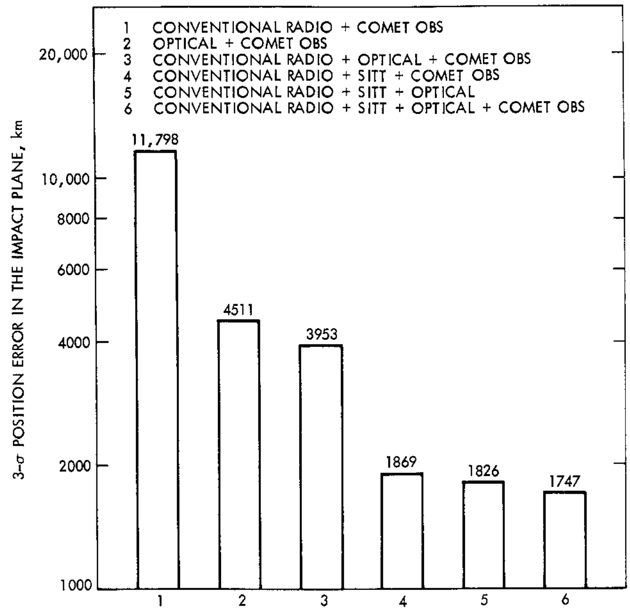

Fig. 7 Relative effectiveness of various data type combinations in reducing delivery errors for the baseline case.

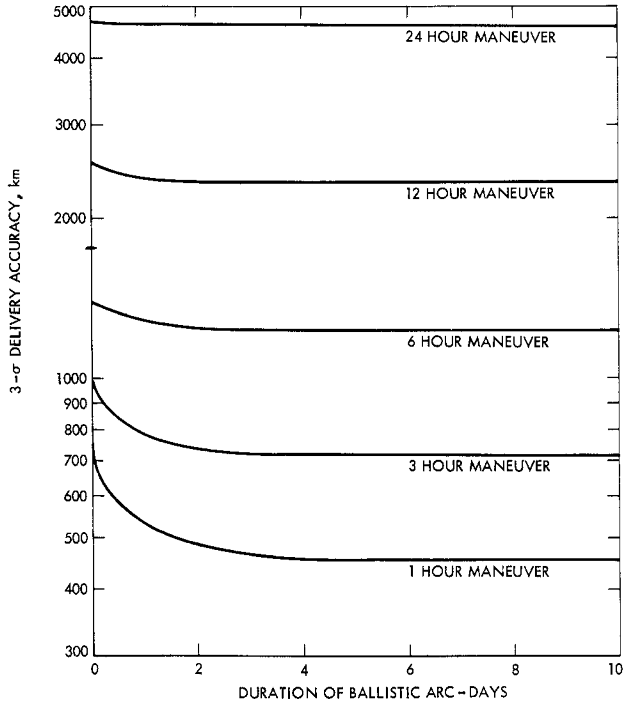

Fig. 8 Delivery accuracy vs ballistic arc duration for several final maneuver times.

evaluate the effectiveness of this procedure, the baseline navigation strategy was modified to include a ballistic coast arc of up to 10 days duration, followed by a final low thrust maneuver of 1 to 24 hrs. In each case, the final maneuver is assumed to be implemented using the full thrust level available. For the longer maneuver times, where less than full thrust is needed to null the projected delivery error, the additional component of thrust is assumed to be applied to a time of flight correction. Because of transients associated with thruster restarts, the standard deviation of the short term thrust noise during the final maneuver was set at 5.0%. Figure 8 shows the delivery accuracy attainable with various combinations of coast and final maneuver times, and Fig. 9 gives

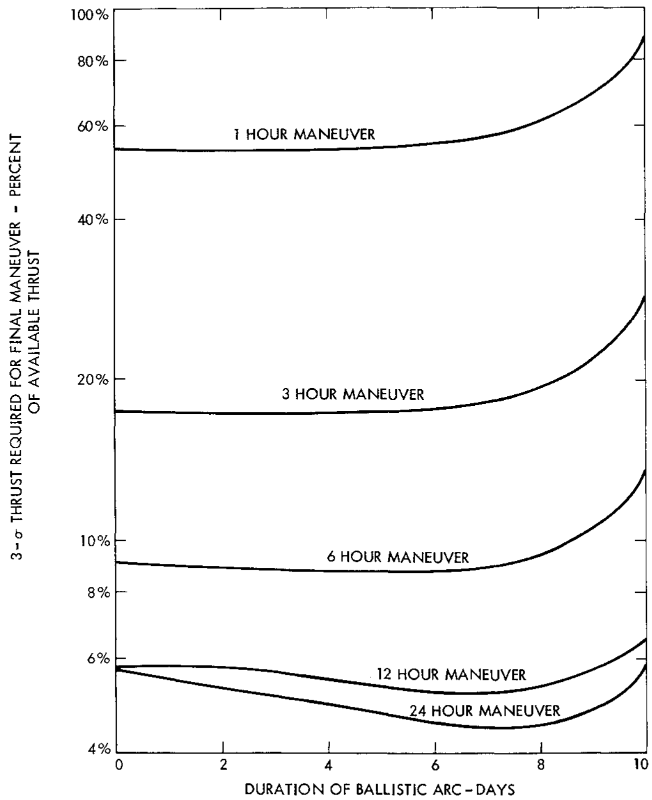

Fig. 9 Final maneuver thrust level as a function of maneuver time and ballistic arc duration.

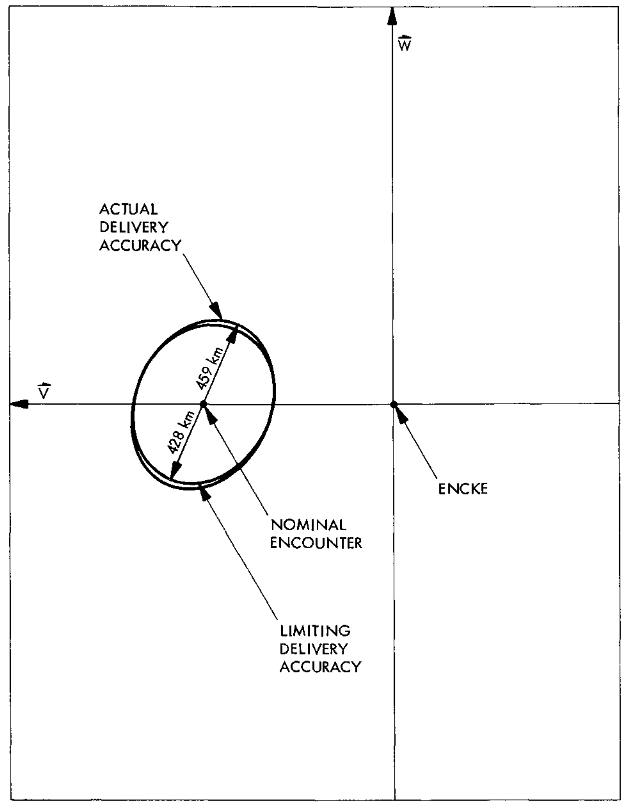

Fig. 10 Delivery error ellipses in the impact plane at encounter for the case of a 4-day ballistic arc followed by a 1-hour final maneuver.

the percent of available thrust required to achieve the indicated accuracy.

The addition of a coast results in a significant improvement in knowledge of the spacecraft state; however, as the length of the coast is increased, the rate of improvement diminishes rapidly because the limits imposed by the remaining error sources are approached. Since the final maneuver is performed at a high thruster noise level (due to the restart), new errors are introduced, which prevent the delivery accuracy from reaching the level of the knowledge or orbit determination accuracy. From Fig. 8 it is evident that the noise effect during a 12 or 24 hr maneuver negates all benefits derived from the coast and even produces delivery errors greater than those occurring without a coast. Shortening the maneuver time reduces the noise effect, and allows the knowledge gain from the coast to be transformed into a corresponding improvement in delivery. In all cases considered, sufficient thrust is available to execute the desired final maneuver. The required thrust level increases as the coast lengthens and the maneuver time decreases. The reasons for the increase are clear. First, a longer coast leads to larger uncorrected errors, but improved knowledge; thus, the maneuver which nulls the known error is necessarily larger. Second, a shorter maneuver time implies the need for a large maneuver to achieve and equivalent control correction capability.

Figure 8 shows that the required 500 km delivery accuracy can be met with a 1-hr maneuver if a coast of at least 1.8 days is employed. For coasts longer than 4 days, little additional accuracy is gained, but the thrust control requirement increases, as indicated in Fig. 9. Therefore, the navigation strategy selected for more detailed analysis is that which employs a 4-day coast followed by a 1-hr final maneuver. For this case, the impact plane ellipses analogous to those of Fig. 5 are displayed in Fig. 10. Note that the actual delivery is quite close to the limiting delivery accuracy, indicating the small effect of thrust vector noise during the 1-hr maneuver. A comparison of the time history of the projected delivery error and of the knowledge error (i.e., the orbit determination error

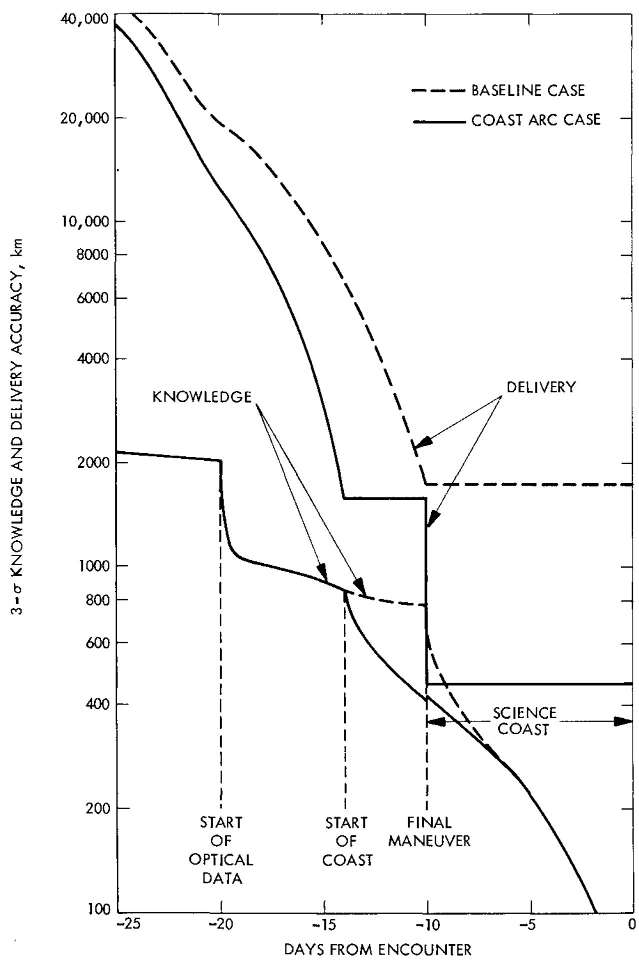

Fig. 11 Time history of knowledge and delivery errors for the baseline case and selected coast case.

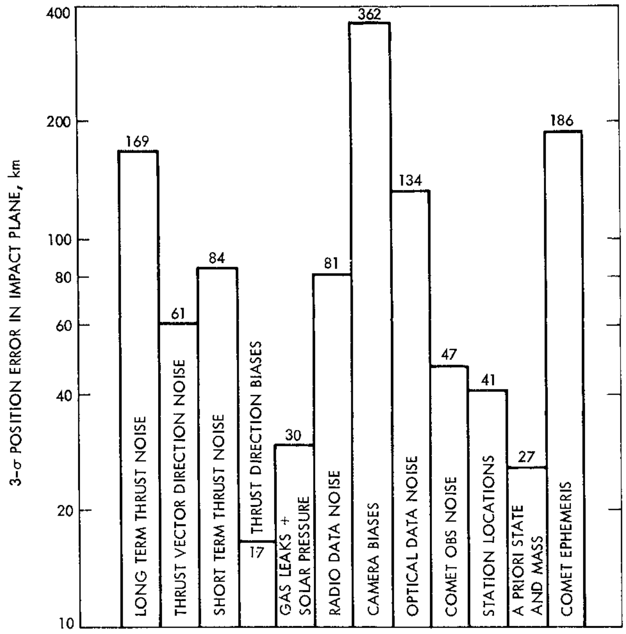

Fig. 12 Relative contribution of the various navigation error sources to delivery error for the selected coast case.

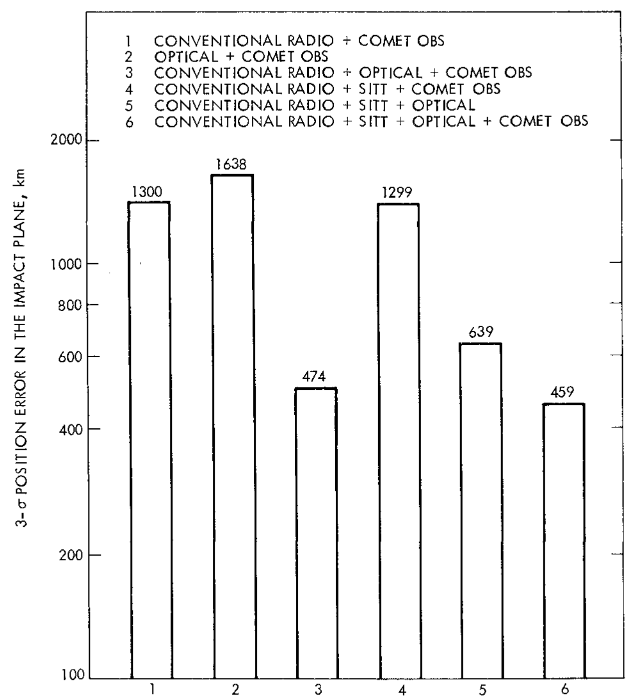

Fig. 13 Relative effectiveness of various data type combinations in reducing delivery errors for the selected coast case.

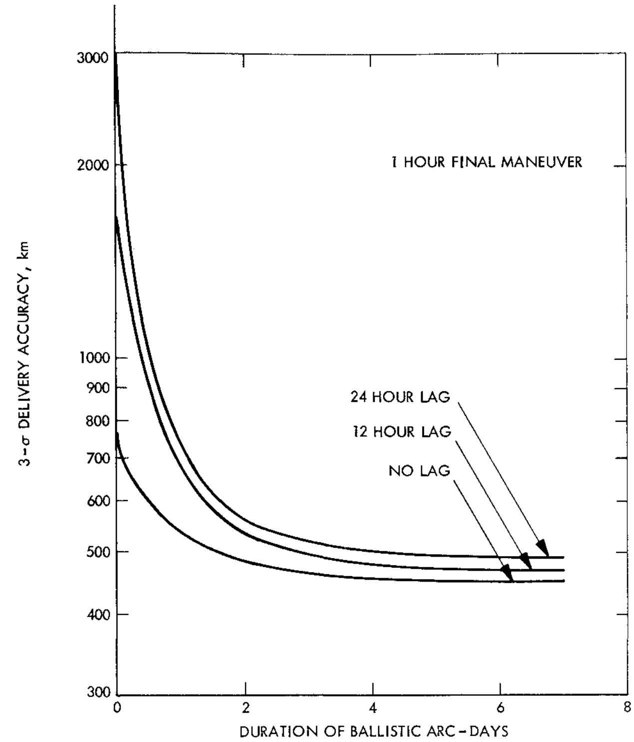

Fig. 14 Delivery accuracy vs ballistic arc duration for various information lags.

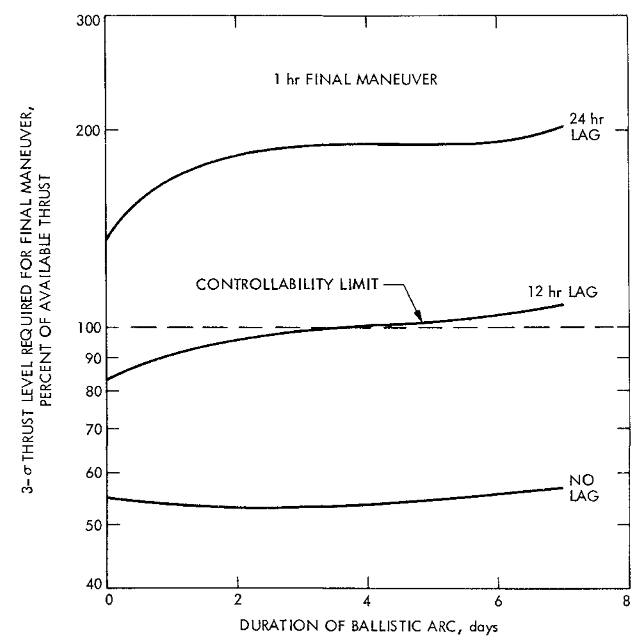

Fig. 15 Final maneuver thrust level as a function of ballistic arc duration for various information lags.

mapped to encounter) for the baseline case and the coast case is given in Fig. 11. Note that the knowledge improves dramatically with the introduction of the optical data and also during the coast. Even more dramatic is the projected delivery improvement immediately following the final maneuver in the coast case.

Comparison of the contributions of the various error sources for the coast case (Fig. 12) with the baseline case (Fig. 6) shows that the effects of all but one of the sources have been reduced. The relative contribution of the gas leaks and solar pressure has increased because those disturbances become significant error sources in the absence of thruster noise (i.e., during a coast). Comparison of the effectiveness of the various data type combinations for the coast case (Fig. 13) with the baseline case (Fig. 7) shows that with the coast all combinations perform better than the best performance (including all data types) without the coast. Moreover, the importance of the dual station radio data (SITT) has diminished, and that of the optical data has increased significantly. Referring again to Figs. 6 and 12, the importance of the optical data is also reflected in the nearly unchanged contribution of the optical data noise and camera biases to the delivery error, while the reduced contribution of the radio data noise reflects the lesser role of the radio data. Finally, it should be noted that observations involving all three legs of

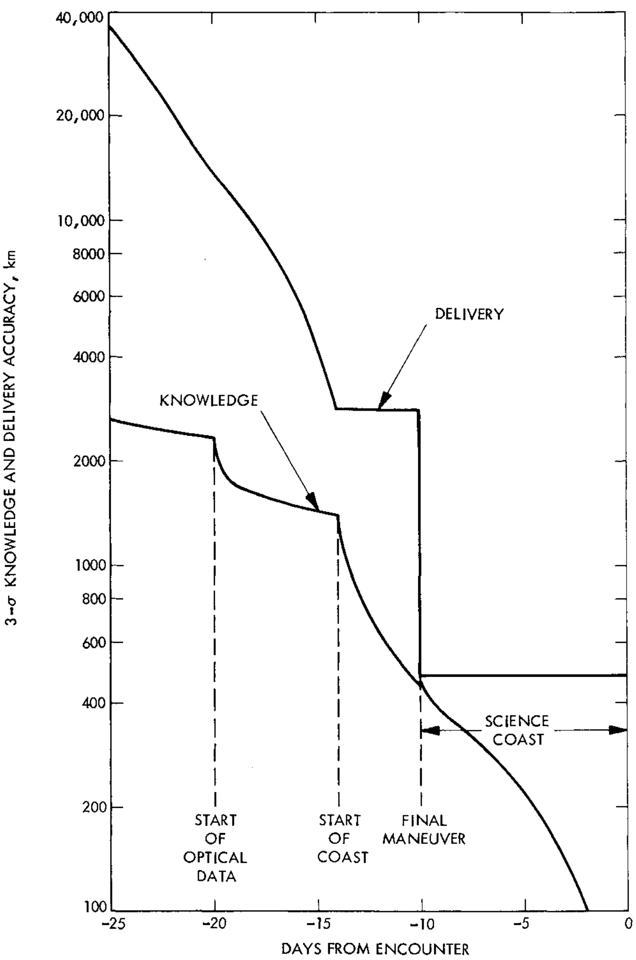

Fig. 16 Time history of knowledge and delivery errors for the case of a 4 day coast, 1 hour final maneuver, and a 12 hour information lag.

the "triangle" formed by the Earth, comet, and spacecraft are necessary to achieve an acceptable delivery accuracy even when the coast arc is included.

V. Effects of an Information Lag

All of the analysis presented thus far has assumed an "instantaneous" control update, i.e., guidance commands are based on the current state estimate. Actual operations impose a time delay or information lag between the orbit determination function and the trajectory control update function and force control corrections to be determined on the basis of earlier state estimates mapped ahead to the time of implementation of the next correction. Because of errors inherent in the prediction process, a degradation in delivery accuracy can be expected. Moreover, the loss of accuracy will increase with the size of the lag, since longer lags require predictions over greater time periods and permit additional error growth. In Fig. 14 delivery as a function of information lag is shown for various coast lengths followed by a 1-hr maneuver. For no coast or for a very short coast, a large delivery error occurs; but when the length of the coast is increased, the effect of the lag diminishes sharply. Clearly, when there is no coast or when the coast is short relative to the information lag, the presence of the process noise degrades the state estimate prediction, resulting in large errors. The addition of a longer coast permits the final maneuver to be based on predictions made in the absence of thrust vector noise. Consequently, the quality of the prediction is vastly improved, and the delivery accuracy increases accordingly.

Unfortunately, Fig. 14 does not represent the entire effect of an information lag on navigation in the presence of a coast arc. From Fig. 15 it can be seen that much larger thrust levels are required to produce the delivery accuracies of Fig. 14. In fact, the limit of controllability (i.e., greater than 100% available thrust required) is exceeded for coasts longer than 4 days when a 12-hr lag is present, and for all coasts when the lag is 24 hrs. The reason for the large thrust levels is the poorer trajectory control before the coast, which leaves larger errors to be removed by the final maneuver. When the time history of the delivery and knowledge for the 4-day coast with a 12-hr lag, Fig. 16, is compared with the no lag case, Fig. 11, the poorer delivery and knowledge, prior to the coast are evident. Since the coast virtually eliminates the effect of the lag on knowledge, the large projected delivery error at the final maneuver time in the lagged case is known as well as the smaller error in the unlagged case. Consequently, a larger maneuver is commanded in the attempt to drive the delivery error to the knowledge level.

Because of the degrading effect of the thruster noise on state estimate prediction, it is clear that operational delays between orbit determination and updated control implementation must be minimized for effective navigation. The most troublesome effect of a lag is a decrease in controllability at the final maneuver time. The addition of a navigation coast reduces prediction errors and will permit acceptable delivery accuracies to be achieved for lags at least as long as 12 hr.

VI. Conclusions

The baseline navigation strategy for the proposed Encke Slow Flyby Mission cannot achieve the required 500 km delivery accuracy because thrust vector noise prevents adequate orbit determination during powered flight. However, if a navigation coast is included in the mission profile, errors in the estimated spacecraft state are significantly reduced, and the required delivery accuracy can be achieved with a final brief low thrust maneuver. The coast should be long enough to permit substantial knowledge enhancement, but short enough to prevent excessive uncontrolled error propagation, and the final maneuver should be as short as possible to reduce trajectory corruption by the thrust vector noise. Tracking strategies during the terminal navigation phase must include Earth-based measurements of both the spacecraft and the comet, as well as onboard optical observations of the comet by the spacecraft. However, the presence of a navigation coast reduces the need for dual station radio tracking of the spacecraft. This does not imply that dual station tracking will not be required for the mission, since it almost certainly will be needed for effective cruise navigation and would therefore be continued routinely into the terminal phase.

In general, a time delay between orbit determination and implementation of trajectory control updates severely degrades delivery accuracy because the control updates must be based on the preduction of future states along a powered trajectory. However, the navigation coast allows predictions for the last maneuver to be made along an unpowered trajectory, and restores the delivery accuracy to an acceptable level. As a consequence of the poorer flight path control prior to the coast, the magnitude of the required final maneuver increases in the presence of a delay, and in fact may exceed the thrust level available. Further work is necessary to identify the best selection of coast duration and final maneuver time as a function of the time delay, and to determine acceptable limits on the size of the delay.

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